

Proposta de resolução

Grupo I

1. (b_n) é uma progressão aritmética, pois:

$$b_{n+1} - b_n = \ln(a_{n+1}) - \ln(a_n) = \ln\left(\frac{a_{n+1}}{\underbrace{a_n}_{(a_n) \text{ p.g.}}}\right) = \ln(e) = 1$$

A razão da progressão aritmética é igual a 1

$$b_1 = \ln 2$$

$$b_n = b_1 + (n - 1) \times r \Leftrightarrow b_n = \ln 2 + n - 1$$

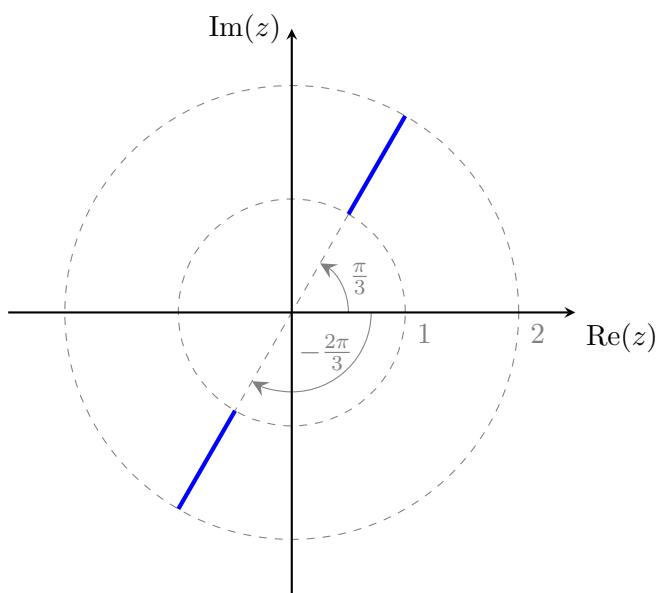
$$b_{10} = \ln 2 + 9$$

$$S_{10} = \frac{b_1 + b_{10}}{2} \times 10 = \frac{\ln 2 + \ln 2 + 9}{2} \times 10 = (\ln 4 + \ln e^9) \times 5 = 5 \ln(4 \cdot e^9)$$

Opção (B)

2.

$$\begin{aligned} \left| \frac{\pi}{3} + \operatorname{Arg}(z) \right| = \pi &\Leftrightarrow \frac{\pi}{3} + 2 \operatorname{Arg}(z) = \pi \quad \vee \quad \frac{\pi}{3} + 2 \operatorname{Arg}(z) = -\pi \Leftrightarrow \\ &\Leftrightarrow 2 \operatorname{Arg}(z) = \frac{2\pi}{3} \quad \vee \quad 2 \operatorname{Arg}(z) = -\frac{4\pi}{3} \Leftrightarrow \operatorname{Arg}(z) = \frac{\pi}{3} \quad \vee \quad \operatorname{Arg}(z) = -\frac{2\pi}{3} \\ &1 \leq |z| \leq 2 \quad \wedge \quad \left(\operatorname{Arg}(z) = \frac{\pi}{3} \quad \vee \quad \operatorname{Arg}(z) = -\frac{2\pi}{3} \right) \end{aligned}$$



Opção(B)

3.

$$\frac{P_{[OABC]}}{4} = \overline{OC} \Leftrightarrow \overline{OC} = 4$$

$$C(-4, 0)$$

Seja $\alpha = B\hat{C}D$, então $2\alpha + 2 \times \frac{2\pi}{3} = 2\pi \Leftrightarrow \alpha = \frac{\pi}{3}$

$$A\hat{C}O = \frac{\alpha}{2} = \frac{\pi}{6}$$

$$m_{AC} = \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\therefore y - 0 = \frac{\sqrt{3}}{3}(x + 4) \Leftrightarrow 3y = \sqrt{3}x + 4\sqrt{3} \Leftrightarrow \sqrt{3}x - 3y + 4\sqrt{3} = 0$$

Opção (A)

4.

$$m = \frac{1-0}{0+3} = \frac{1}{3} = \lim_{x \rightarrow +\infty} \frac{g(x)}{x}$$
$$\lim_{x \rightarrow +\infty} h(x) = \lim_{x \rightarrow +\infty} \left(\frac{3x}{g(x)} + 1 \right) = \underbrace{\lim_{x \rightarrow +\infty} \frac{3}{g(x)}}_{\text{declive a.o.}} + 1 = \frac{3}{\frac{1}{3}} + 1 = 9 + 1 = 10$$

A reta de equação $y = 10$ é assíntota horizontal ao gráfico de h .

Opção (B)

5.

$$f(\pi) = k + \ln(2 + \cos \pi) = k + \ln(2 - 1) = k$$

$$\begin{aligned} \lim_{x \rightarrow \pi^-} f(x) &= \lim_{x \rightarrow \pi^-} \frac{\tan x}{\pi - x} \underset{y = \pi - x}{=} \lim_{y \rightarrow 0} \frac{\tan(\pi - y)}{y} = \lim_{y \rightarrow 0} \frac{-\tan y}{y} = \\ &= -\underbrace{\lim_{y \rightarrow 0} \frac{\sin y}{y}}_{\text{limite notável}} \times \lim_{y \rightarrow 0} \frac{1}{\cos y} = -1 \times \frac{1}{1} = -1 \\ \therefore k &= -1 \end{aligned}$$

Opção (A)

6.

$${}^n C_0 + {}^n C_1 + {}^n C_2 + {}^{n+1} C_{n-1} + {}^{n+1} C_n + {}^{n+1} C_{n+1} = 258 \Leftrightarrow 1 + n + \frac{n(n-1)}{2} + \frac{(n+1)n}{2} + n + 1 + 1 = 258 \Leftrightarrow$$

$$3 + 2n + \frac{n^2}{2} - \frac{n}{2} + \frac{n^2}{2} + \frac{n}{2} = 258 \Leftrightarrow n^2 + 2n - 255 = 0 \Leftrightarrow n = 15 \vee n = -17 \Rightarrow n = 15$$



$$\therefore (15 + 1) - 8 = 8$$

Opção (C)

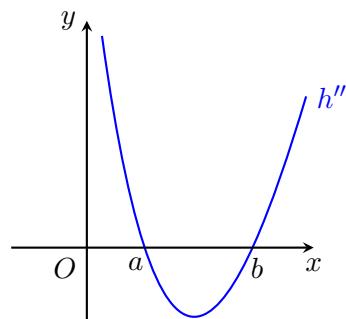
7.

$$\begin{cases} \frac{1}{8} + a + b = 1 \\ 0 \times \frac{1}{8} + 2a + \frac{2b}{a} = \frac{5}{2} \end{cases} \Leftrightarrow \begin{cases} a + b = \frac{7}{8} \\ 4a^2 + 4b = 5a \end{cases} \Leftrightarrow \begin{cases} a - a^2 + \frac{5}{4}a = \frac{7}{8} \\ b = -a^2 + \frac{5}{4}a \end{cases} \Leftrightarrow \begin{cases} -8a^2 + 18a - 7 = 0 \\ b = -a^2 + \frac{5}{4}a \end{cases}$$

$$\Leftrightarrow \begin{cases} a = \frac{1}{2} \\ b = \frac{3}{8} \end{cases} \quad \text{ou} \quad \underbrace{\begin{cases} a = \frac{7}{4} \\ > 1 \end{cases}}_{\Rightarrow} \Rightarrow \begin{cases} a = \frac{1}{2} \\ b = \frac{3}{8} \end{cases}$$

Opção (C)

8.



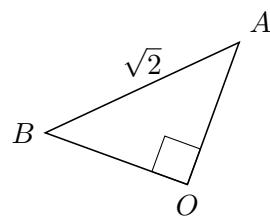
x	0		a		b	$+\infty$
$h''(x)$		+	0	-	0	+
h'		\nearrow	Max.	\searrow	Min.	\nearrow

Opção (D)

Grupo II

1.

$$\overline{AB} = \frac{\sqrt{2}}{4} = \sqrt{2}$$



$$\overline{OA}^2 + \overline{OB}^2 = \sqrt{2}^2 \Leftrightarrow 2\overline{OA}^2 = 2 \Leftrightarrow \overline{OA} = \pm 1 \Rightarrow \overline{AB} = 1 \Leftrightarrow |z_A| = 1$$

$$z_A = e^{i(\frac{7\pi}{18})}$$

$$z = (z_A)^4 = \left[e^{i(\frac{7\pi}{18})} \right]^4 = e^{i\frac{14\pi}{9}} = e^{i(-\frac{4\pi}{9})}$$

$$z_B = z_A \times i = \left(e^{i\frac{7\pi}{18}} \right) \times e^{i\frac{\pi}{2}} = e^{i(\frac{7\pi}{18} + \frac{\pi}{2})} = e^{i\frac{8\pi}{9}}$$

$$w = (z_B)^3 = \left(e^{i\frac{8\pi}{9}} \right)^3 = e^{i\frac{8\pi}{3}} = e^{i\frac{2\pi}{3}} = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z^9 + w = \left[e^{i(-\frac{4\pi}{9})} \right]^9 + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = e^{i(-4\pi)} + -\frac{1}{2} + \frac{\sqrt{3}}{2}i = 1 - \frac{1}{2} + \frac{\sqrt{3}}{2}i = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\left| \frac{1}{2} + \frac{\sqrt{3}}{2}i \right| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$$\text{Arg} \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = \arctan \left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \right) = \arctan \sqrt{3} = \frac{\pi}{3}$$

$$\therefore z^9 + w = e^{i\frac{\pi}{3}}$$

2.

2.1.

$$p = \frac{2 \times 4}{6 \times 4} = \frac{1}{3}$$

2.2.

$$P(B|A) = \frac{3 \times 1 + 3 \times 3}{6 \times 4 - 3 \times 1} = \frac{4}{7}$$

$P(B|A)$ é a probabilidade do produto dos valores obtidos nos dois dados ser negativo, sabendo que o ponto de coordenadas (a, b) não pertence ao primeiro quadrante.

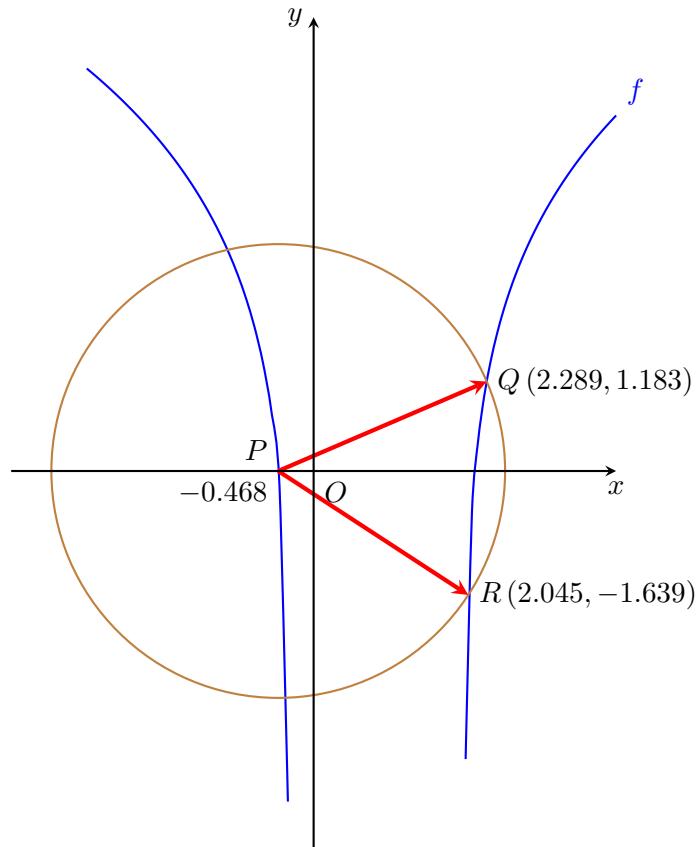
Se o ponto de coordenadas (a, b) não pertence ao primeiro quadrante, então os valores obtidos nos dois dados não são positivos simultaneamente, ou seja, existem $6 \times 4 - 3 \times 1 = 21$ casos possíveis, já que 6×4 representa o total de possibilidades e 3×1 representa o número de casos em que os valores obtidos são ambos positivos.

Para que o produto dos valores obtidos nos dados seja negativo, tem que sair dois números com sinais contrários. Assim, terá que sair um valor negativo no dado cúbico e um valor positivo no dado tetraédrico (3×1 possibilidades) ou sair um valor positivo no dado cúbico e um valor negativo no dado tetraédrico (3×3 possibilidades). Então, o número de casos favoráveis é $3 \times 1 + 3 \times 3 = 12$.

Assim, o valor de $P(B|A) = \frac{12}{21} = \frac{4}{7}$.

3.

3.1.



$$(x + 0.468)^2 + y^2 = 9 \Leftrightarrow y = \pm \sqrt{9 - (x + 0.468)^2}$$

$$\overrightarrow{PQ} = Q - P = (2.757, 1.183)$$

$$\overrightarrow{PR} = R - P = (2.513, -1.639)$$

$$|PQ| = |PR| = 3$$

$$\overrightarrow{PQ} \cdot \overrightarrow{PR} = 2.757 \times 2.513 - 1.183 \times 1.639 = 4.989404$$

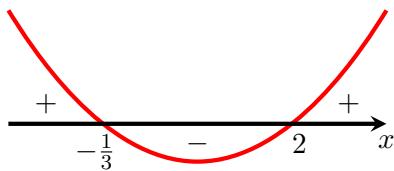
$$\therefore \cos(Q\hat{P}R) = \frac{4.989404}{3 \times 3} \Rightarrow Q\hat{P}R = \arccos\left(\frac{4.898404}{9}\right) \approx 0.98 \text{ rad}$$

3.2.

$$D = \left\{ x \in \mathbb{R} : x - 2 > 0 \wedge 3x^2 - 5x - 2 > 0 \wedge x^2 > 0 \right\} =$$

$$D = \left\{ x \in \mathbb{R} : x > 2 \wedge \left(x > 2 \vee x < -\frac{1}{3}\right) \wedge x \neq 0 \right\} =]2, +\infty[$$

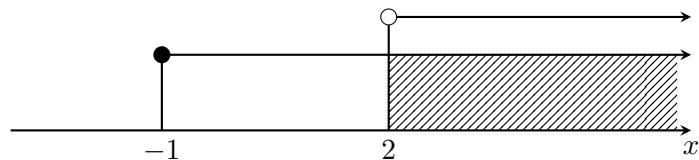
$$3x^2 - 5x - 2 = 0 \Leftrightarrow x = \frac{5 \pm \sqrt{25 - 4 \times 3 \times (-2)}}{6} \Leftrightarrow x = \frac{5 \pm 7}{6} \Leftrightarrow x = 2 \vee x = -\frac{1}{3}$$



$$3x^2 - 5x - 2 = 3 \left(x + \frac{1}{3} \right) (x - 2) = (3x + 1)(x - 2)$$

$$\begin{aligned} 1 + \log_2(x - 2) &\leq \log_2(3x^2 - 5x - 2) - \log_4(x^2) \Leftrightarrow \\ &\Leftrightarrow \log_2 2 + \log_2(x - 2) \leq \log_2[(3x + 1)(x - 2)] - 2 \times \frac{\log_2 x}{\log_2 4} \Leftrightarrow \\ &\Leftrightarrow \log_2 2 + \log_2 x + \log_2(x - 2) \leq \log_2(3x + 1) + \log_2(x - 2) \Leftrightarrow \end{aligned}$$

$$\Leftrightarrow \log_2(2x) \leq \log_2(3x + 1) \Leftrightarrow 2x \leq 3x + 1 \Leftrightarrow -x \leq 1 \Leftrightarrow x \geq -1$$



$$\therefore C.S =]2, +\infty[$$

4.

4.1.

$$\begin{aligned} g'(1) &= \lim_{x \rightarrow 1} \frac{g(x) - g(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{x^2 + 6e^{-x} - 1 - 6e^{-1}}{x - 1} = \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} + 6 \lim_{x \rightarrow 1} \frac{e^{-x} - e^{-1}}{x - 1} = \\ &= \lim_{x \rightarrow 1} \frac{(x - 1)(x + 1)}{x - 1} - \underbrace{\frac{6}{e} \lim_{-x+1 \rightarrow 0} \frac{e^{-x+1}}{-x+1}}_{\text{limite notável}} = 2 - \frac{6}{e} = \frac{2e - 6}{e} \end{aligned}$$

4.2.

$$D = \mathbb{R}$$

$$\begin{aligned} x^2 + e^x = x^2 + 6e^{-x} + 1 &\Leftrightarrow e^x - 1 - \frac{6}{e^x} = 0 \Leftrightarrow e^{2x} - e^x - 6 = 0 \Leftrightarrow e^x = \frac{1 \pm \sqrt{1 - 4 \times 1 \times (-6)}}{2} \Leftrightarrow \\ &\Leftrightarrow e^x = \frac{1 \pm 5}{2} \Leftrightarrow e^x = 3 \vee \underbrace{e^x = -2}_{\text{eq. imp.}} \Leftrightarrow x = \ln 3 \end{aligned}$$

$$\therefore S = \{\ln 3\}$$

5.

5.1. $\vec{n}(5, -1, 3)$ é um vetor diretor da reta.

$$C(0, 3, 0)$$

$$\therefore (x, y, z) = (0, 3, 0) + k(5, -1, 3), \quad k \in \mathbb{R}$$

5.2.

$$V(0, 0, z)$$

$$5 \times 0 - 0 + 3z = 12 \Leftrightarrow z = 4$$

$$V(0, 0, 4)$$

$$A(x, -2, 0)$$

$$5x + 2 + 3 \times 0 = 12 \Leftrightarrow x = 2$$

$$A(2, -2, 0)$$

$$B(x, 3, 0)$$

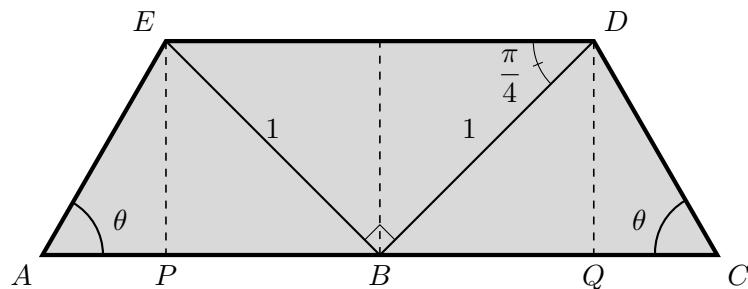
$$5x - 3 + 3 \times 0 = 12 \Leftrightarrow x = 3$$

$$B(3, 3, 0)$$

$$\therefore V = \frac{1}{3} A_b \times h = \frac{1}{3} \times \frac{3+2}{2} \times 5 \times 4 = \frac{50}{3}$$

6.

6.1.



$$\sin \frac{\pi}{4} = \frac{\overline{DQ}}{1} \Leftrightarrow \overline{DQ} = \frac{\sqrt{2}}{2}$$

$$\tan \theta = \frac{\frac{\sqrt{2}}{2}}{\overline{QC}} \Leftrightarrow \overline{QC} = \frac{\sqrt{2}}{2 \tan \theta}$$

$$\therefore A_{[ACDE]} = 2 \times \left[\left(\frac{\sqrt{2}}{2} \right)^2 + \frac{\sqrt{2}}{2 \tan \theta} \times \frac{\sqrt{2}}{2} \right] = 2 \left(\frac{1}{2} + \frac{1}{4 \tan \theta} \right) = 1 + \frac{1}{2 \tan \theta}$$

6.2.

$$A'(\theta) = \left(1 + \frac{1}{2\theta} \right)' = 0 + \frac{0 - 1 \times \frac{2}{\cos^2 \theta}}{4 \tan^2 \theta} = -\frac{2}{4 \times \frac{\sin^2 \theta}{\cos^2 \theta}} = -\frac{1}{2 \sin^2 \theta}$$

Como $\sin^2 \theta > 0$, $\forall \theta \in \left]0, \frac{\pi}{2}\right[$, então $A'(\theta) = -\frac{1}{2 \sin \theta} < 0$, $\forall \theta \in \left]0, \frac{\pi}{2}\right[$.

Portanto, A é estritamente decrescente em $\left]0, \frac{\pi}{2}\right[$.

6.3.

$$\lim_{x \rightarrow \frac{\pi}{2}^-} A(\theta) = \lim_{x \rightarrow \frac{\pi}{2}^-} \left(1 + \frac{1}{2 \tan \theta} \right) = 1 + \frac{1}{2 \tan \frac{\pi}{2}^-} = 1 + \frac{1}{+\infty} = 1 + 0 = 1$$

É a área de um retângulo de dimensões $\sqrt{2}$ por $\frac{\sqrt{2}}{2}$.