

Proposta de resolução

Grupo I

1.

$$r = \frac{a_2}{a_1} = \frac{-\frac{2}{5}}{-\frac{3}{5}} = \frac{2}{3}$$

$$a_n = a_1 \times r^{n-1} \Leftrightarrow a_n = -\frac{3}{5} \times \left(\frac{2}{3}\right)^{n-1}$$

$$\lim(a_n) = \lim \left[-\frac{3}{5} \times \left(\frac{2}{3}\right)^{n-1} \right] = -\frac{3}{5} \times \left(\frac{2}{3}\right)^{+\infty} = -\frac{3}{5} \times 0^+ = 0^-$$

$$\lim f(a_n) = f(0^-) = 1$$

Opção (A)

2.

$$g(-2) = -g(2) = \frac{1}{2}$$

$$\lim_{x \rightarrow -2} \frac{g(x) + g(2)}{x + 2} = 3 \Leftrightarrow \underbrace{\lim_{x \rightarrow -2} \frac{g(x) - g(-2)}{x - (-2)}}_{\text{definição de derivada}} = 3 \Leftrightarrow g'(-2) = 3$$

$$f'(x) = 2e^{2x-1}$$

$$f'\left(\frac{1}{2}\right) = 2e^{2 \times \frac{1}{2} - 1} = 2e^0 = 2$$

$$(f \circ g)'(-2) = f'(g(-2)) \times g'(-2) = f'\left(\frac{1}{2}\right) \times 3 = 2 \times 3 = 6$$

$$f(x) = e - 4 \Leftrightarrow e^{2x-1} - 4 = e - 4 \Leftrightarrow e^{2x-1} = e \Leftrightarrow 2x - 1 = 1 \Leftrightarrow x = 1$$

Assim, $f^{-1}(e - 4) = 1$

$$\therefore (f \circ g)'(-2) - f^{-1}(e - 4) = 6 - 1 = 5$$

Opção (B)

3. f é contínua em $x = 0$ se e só se $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = f(0)$.

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{e^{2x} + x - 1}{x - \sqrt{2x}} = \lim_{x \rightarrow 0^+} \frac{e^{2x} + x - 1}{x} \times \lim_{x \rightarrow 0^+} \frac{x}{x - \sqrt{2x}} =$$

$$= \left(2 \underbrace{\lim_{2x \rightarrow 0^+} \frac{e^{2x} - 1}{2x}}_{\text{limite notável}} + \lim_{x \rightarrow 0^+} \frac{x}{x} \right) \times \lim_{x \rightarrow 0^+} \frac{x(x + \sqrt{2x})}{x^2 - 2x} = (2 \times 1 + 1) \times \lim_{x \rightarrow 0^+} \frac{x + \sqrt{2x}}{x - 2} = 3 \times \frac{0 + 0}{0 - 2} = 0$$

$$f(0) = \cos(3 \times 0 - k) = \cos(-k)$$

$$\cos(-k) = 0 \Leftrightarrow -k = \frac{\pi}{2} + \lambda\pi, \lambda \in \mathbb{Z} \Leftrightarrow k = -\frac{\pi}{2} - \lambda\pi, \lambda \in \mathbb{Z}$$

$$\text{Se } \lambda = -1 \Rightarrow k = \frac{\pi}{2}$$

Opção (C)

4.

$$\begin{aligned} P(A \cap B) &= 0 \\ P(B) &= P(\overline{A} \cap B) + P(A \cap B) = 0.55 + 0 = 0.55 \end{aligned}$$

$$P(\overline{A}) = P(\overline{A} \cap B) + P(\overline{A} \cap \overline{B}) \Leftrightarrow 0.7 = 0.55 + P(\overline{A} \cap \overline{B}) \Leftrightarrow P(\overline{A} \cap \overline{B}) = 0.15$$

$$\therefore P(\overline{A}|\overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{0.15}{1 - 0.55} = \frac{1}{3}$$

Opção (D)

5.

$$\frac{^{2013}C_{100}}{^{2015}C_{102}} + \frac{^{2013}C_{1912}}{^{2015}C_{102}} + \frac{a}{^{2015}C_{102}} = 1 \Leftrightarrow ^{2013}C_{100} + \underbrace{^{2013}C_{101}}_{^nC_p = ^nC_{n-p}} + a = ^{2015}C_{102} \Leftrightarrow ^{2014}C_{101} + a = ^{2015}C_{102}$$

Assim, $a = ^{2014}C_{102}$, pois $^nC_p + ^nC_{p+1} = ^{n+1}C_{p+1}$.

Opção (D)

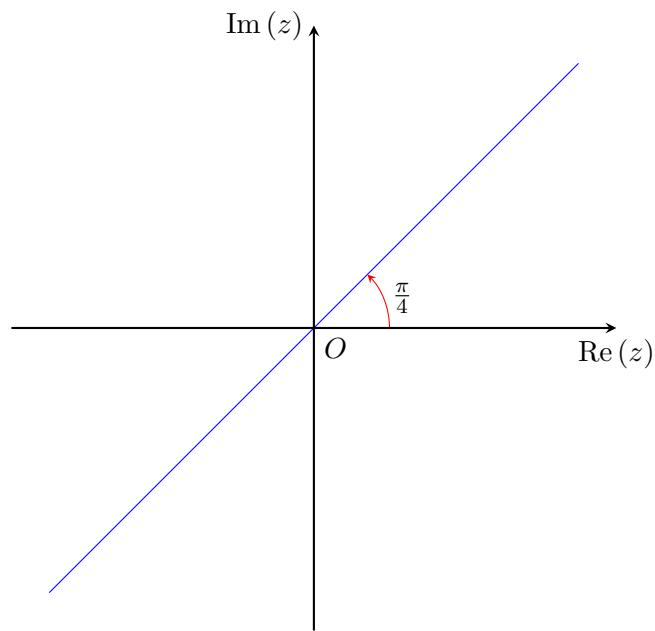
6.

$$i^{4k+3} = i^3 = -i = e^{i(-\frac{\pi}{2})}, k \in \mathbb{N}_0$$

$$|1+i| = \sqrt{2}$$

$$\text{Arg}(1+i) = \arctan(1) = \frac{\pi}{4}$$

$$z = (1+i)^4 \times \left[\frac{i^{4k+3}}{e^{i\alpha}} \right]^2 = \left(\sqrt{2}e^{i\frac{\pi}{4}} \right)^4 \times \left[\frac{e^{i(\frac{\pi}{2})}}{e^{i\alpha}} \right]^4 = 4e^{i\pi} \times \left[e^{i(-\frac{\pi}{2}-\alpha)} \right]^2 = 4e^{i\pi} \times e^{i(\pi-2\alpha)} = 4e^{i(-2\alpha)}$$



Para a imagem geométrica de z pertencer à bissetriz dos quadrantes ímpares, tem que:

$$-2\alpha = \frac{\pi}{4} + k\pi, \quad k \in \mathbb{Z} \Leftrightarrow \alpha = -\frac{\pi}{8} - k\frac{\pi}{2}, \quad k \in \mathbb{Z}$$

$$\text{Se } k = -1 \Rightarrow \alpha = \frac{3\pi}{8}$$

Opção (A)

7.

$$f(a) = 3$$

$$f'(a) = m_t = \tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$$

$$f''(a) < 0$$

$$f(a) \times f'(a) + f''(a) = 3 \times \frac{\sqrt{3}}{3} + \underbrace{f''(a)}_{<0} < \sqrt{3}$$

Opção (B)

8.

$$\begin{aligned} \lim_{x \rightarrow +\infty} \left[g(-x) \times \frac{x}{g(x)} + x - g(x) \right] &= \lim_{x \rightarrow +\infty} g(-x) \times \lim_{x \rightarrow +\infty} \frac{x}{g(x)} + \lim_{x \rightarrow +\infty} (x - g(x)) = \\ &= g(-\infty) \times \underbrace{\lim_{x \rightarrow +\infty} \frac{g(x)}{x}}_{\substack{\text{declive a.o.} \\ \text{ordenada na origem a.o.}}} - \underbrace{\lim_{x \rightarrow +\infty} (g(x) - x)}_{\substack{\text{ordenada na origem a.o.}}} = 1 \times \frac{1}{1} - 1 = 0 \end{aligned}$$

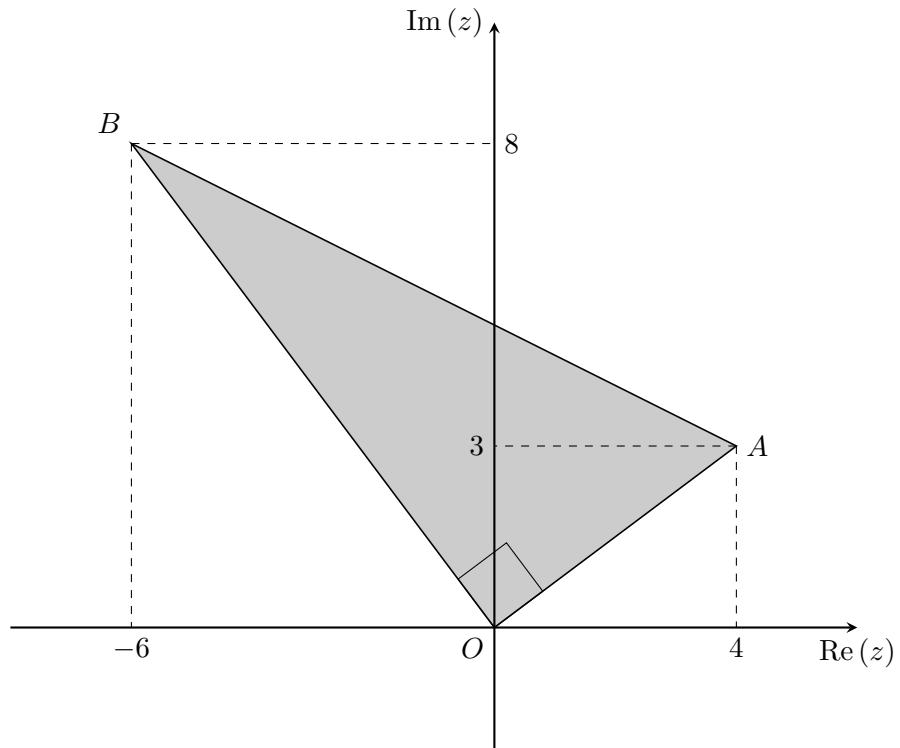
Opção (C)

Grupo II

1.

$$z_1 = \frac{(1+2i)^2}{\left[e^{i(\frac{\pi}{4})}\right]^{10}} = \frac{1+4i-4}{e^{i(\frac{5\pi}{2})}} = \frac{-3+4i}{i} = \frac{(-3+4i)i}{-1} = 4+3i$$

$$z_2 = 2i \times z_1 = 2i(4+3i) = 8i + 6i^2 = -6 + 8i$$



$$A_{[OAB]} = \frac{\overline{OA} \times \overline{OB}}{2} = \frac{\sqrt{4^2 + 3^2} \times \sqrt{(-6)^2 + 8^2}}{2} = \frac{5 \times 10}{2} = 25$$

2.

2.1. Sejam os acontecimentos:

- A : “ser rapaz”
- B : “pretende seguir um curso de engenharia”

$$P(\bar{A}) = 0.4$$

$$P(A \cap B) = 0.3$$

$$P(\bar{B}|\bar{A}) = \frac{2}{3} \Leftrightarrow P(\bar{A} \cap \bar{B}) = \frac{2}{3} \times 0.4 = \frac{4}{15} \Leftrightarrow P(\bar{A} \cup \bar{B}) = \frac{4}{15} \Leftrightarrow P(A \cup B) = \frac{11}{15} \Leftrightarrow$$

$$P(A) + P(B) - P(A \cap B) = \frac{11}{15} \Leftrightarrow 0.6 + P(B) - 0.3 = \frac{11}{15} \Leftrightarrow P(B) = \frac{13}{30}$$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{\frac{13}{30}} = \frac{9}{13}$$

2.2.

$$30 \times 0.4 = 12 \text{ raparigas}$$

$$30 - 12 = 18 \text{ rapazes}$$

$\therefore {}^{30}C_4 - {}^{18}C_4 = 24345$ comissões nas condições pretendidas.

3.

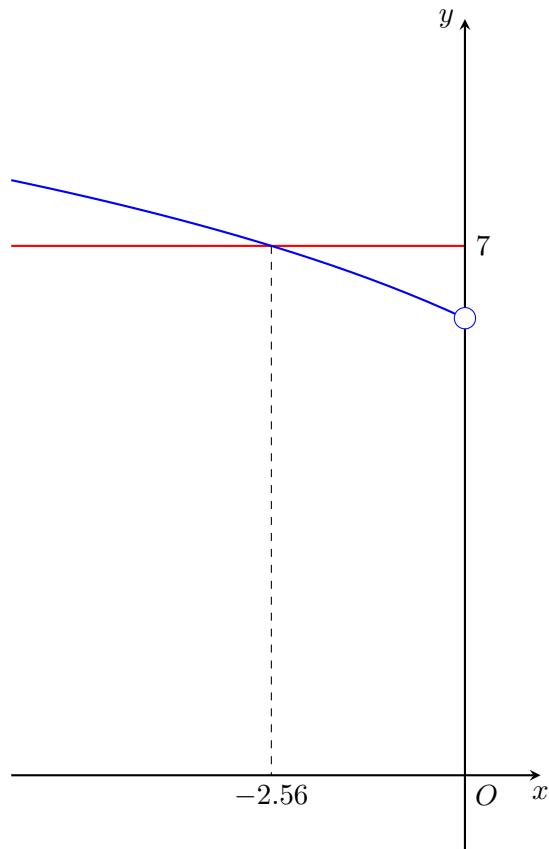
$$f(x) = 1 \Leftrightarrow -\ln(5-x) = 1 \Leftrightarrow 5-x = e^{-1} \Leftrightarrow x = 5 - \frac{1}{e} \Leftrightarrow x = \frac{5e-1}{e} = \overline{AB}$$

Seja x a abcissa do ponto C , então:

$$h = 1 + |f(x)| = 1 + |- \ln(5-x)| = 1 + \ln(5-x) \quad (x < 0)$$

$$A_{[ABC]} = \frac{\overline{AB} \times h}{2} = \frac{\frac{5e-1}{e} \times (1 + \ln(5-x))}{2} = \frac{5e-1}{2e} (1 + \ln(5-x))$$

$$A_{[ABC]} = 7 \Leftrightarrow \frac{5e-1}{2e} (1 + \ln(5-x)) = 7$$



$$f(-2.56) = -\ln(5+2.56) \approx -2.02$$

$$\therefore C(-2.56, -2.02)$$

4.

4.1.

$$\begin{aligned} \lim_{x \rightarrow \frac{3\pi}{2}} \frac{g\left(\frac{3\pi}{2}\right) - g(x)}{2x - 3\pi} &= -\frac{1}{2} \lim_{x \rightarrow \frac{3\pi}{2}} \underbrace{\frac{g(x) - g\left(\frac{3\pi}{2}\right)}{x - \frac{3\pi}{2}}}_{\text{definição de derivada}} = -\frac{1}{2} \times g'\left(\frac{3\pi}{2}\right) = -\frac{1}{2} \left(e^{\sin(\frac{3\pi}{2})} + \cos(3\pi)\right) = \\ &= -\frac{1}{2} \left(e^{-1} - 1\right) = \frac{1}{2} \left(1 - \frac{1}{e}\right) = \frac{e-1}{2e} \end{aligned}$$

4.2.

$$g''(x) = \cos x e^{\sin x} - 2 \sin(2x)$$

g'' é contínua em $\left[\frac{3\pi}{4}, \pi\right]$ pois é a diferença entre funções contínuas.

$$g''\left(\frac{3\pi}{4}\right) = \cos \frac{3\pi}{4} e^{\sin \frac{3\pi}{4}} - 2 \sin \frac{3\pi}{2} = -\frac{\sqrt{2}}{2} e^{\frac{\sqrt{2}}{2}} + 2 > 0$$

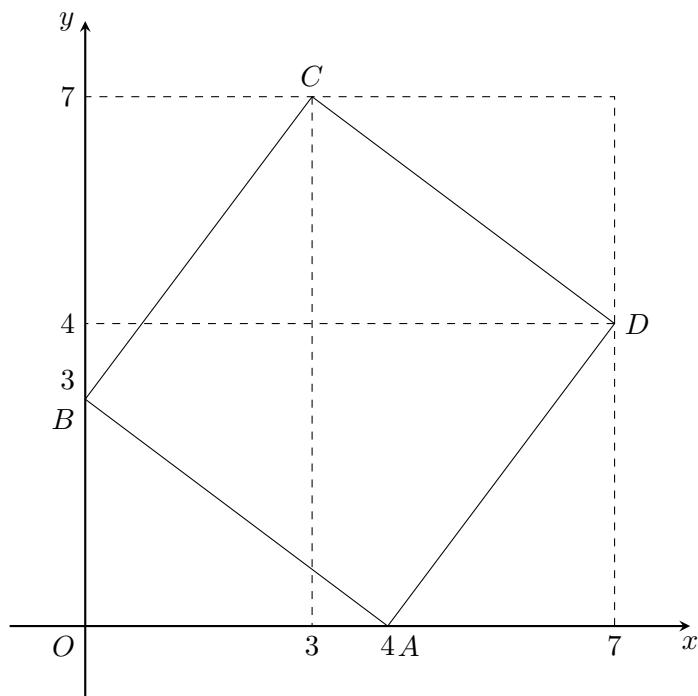
$$g''(\pi) = \cos \pi e^{\sin \pi} - 2 \sin 2\pi = -e^0 - 2 \times 0 = -1 < 0$$

Como $g''\left(\frac{3\pi}{4}\right) \times g''(\pi) < 0$ e g'' é contínua em $\left[\frac{3\pi}{4}, \pi\right]$, então pelo corolário do teorema de Bolzano, $g''(x) = 0$ é possível em $\left]\frac{3\pi}{4}, \pi\right[$ e como existe mudança de sinal de g'' em $\left[\frac{3\pi}{4}, \pi\right]$, então o gráfico de g admite pelo menos um ponto de inflexão com abcissa $a \in \left]\frac{3\pi}{4}, \pi\right[$.

5.

5.1.

$$\begin{aligned} A(4, 0, 0) \\ B(0, 3, 0) \\ \overline{AB} = \sqrt{4^2 + 3^2 + 0^2} = 5 \end{aligned}$$



$$\overrightarrow{AD} = (a, b, 0)$$

$$\overrightarrow{AB} = (-4, 3, 0)$$

$$\overrightarrow{AD} \cdot \overrightarrow{AB} = 0 \Leftrightarrow -4a + 3b = 0 \Leftrightarrow b = \frac{4}{3}a$$

$$\overline{AD} = 5 \Leftrightarrow \sqrt{a^2 + b^2 + 0^2} = 5 \Rightarrow a^2 + \frac{16}{9}a^2 = 25 \Leftrightarrow a^2 = 9 \underbrace{\Rightarrow}_{a>0} a = 3$$

Assim, $\overrightarrow{AD} = (3, 4, 0)$

$$D = A + \overrightarrow{AD} = (4, 0, 0) + (3, 4, 0) = (7, 4, 0)$$

$$C = D + \overrightarrow{AB} = (7, 4, 0) + (-4, 3, 0) = (3, 7, 0)$$

$$\therefore G(3, 7, 5)$$

$$\overrightarrow{DE} = (-3, -4, 5)$$

$$\overrightarrow{DG} = (-4, 3, 5)$$

$$\therefore \overrightarrow{DE} \cdot \overrightarrow{DG} = -3 \times (-4) + (-4) \times 3 + 5 \times 5 = 25$$

5.2. Seja $\vec{n} = (a, b, c)$ um vetor normal ao plano α .

$$\begin{cases} \vec{n} \cdot \overrightarrow{DE} = 0 \\ \vec{n} \cdot \overrightarrow{DG} = 0 \end{cases} \Leftrightarrow \begin{cases} -3a - 4b + 5c = 0 \\ -4a + 3b + 5c = 0 \end{cases} \Leftrightarrow \begin{cases} -3a - 4b + 4a - 3b = 0 \\ 5c = 4a - 3b \end{cases} \Leftrightarrow \begin{cases} a = 7b \\ c = 5b \end{cases}$$

Fazendo $b = 1$, por exemplo, tem-se $\vec{n} = (7, 1, 5)$

$$\therefore 7(x - 7) + 1(y - 4) + 5(z - 0) = 0 \Leftrightarrow 7x + y + 5z = 53$$

5.3.

$$\overrightarrow{OF} = (7, 4, 5)$$

$$OF : (x, y, z) = (0, 0, 0) + k(7, 4, 5), \quad k \in \mathbb{R} \Leftrightarrow (x, y, z) = (7k, 4k, 5k), \quad k \in \mathbb{R}$$

$$7x + y + 5z = 53 \Rightarrow 7 \times 7k + 4k + 5 \times 5k = 53 \Leftrightarrow k = \frac{53}{78}$$

\therefore O ponto de interseção é $\left(\frac{371}{78}, \frac{106}{39}, \frac{265}{78}\right)$

6.

6.1.

$$\tan\left(\alpha - \frac{\pi}{2}\right) = \frac{\overline{BC}}{1} \Leftrightarrow \overline{BC} = -\frac{1}{\tan \alpha}$$

$$\begin{aligned} A_{\text{Sombreada}} &= A_{[OBC]} - A_{\text{Setor circular}} = \frac{\overline{OB} \times \overline{BC}}{2} - \frac{1}{2} \left(\alpha - \frac{\pi}{2}\right) \times 1^2 = \frac{1 \times \frac{-1}{\tan \alpha}}{2} - \frac{1}{2} \alpha + \frac{\pi}{4} = \\ &= -\frac{1}{\tan \alpha} - \frac{1}{2} \alpha + \frac{\pi}{4} = \frac{1}{2} \left(\frac{\pi}{2} - \alpha - \frac{1}{\tan \alpha}\right) \end{aligned}$$

6.2.

$$\lim_{\alpha \rightarrow \pi^-} A(\alpha) = \lim_{\alpha \rightarrow \pi^-} \left[\frac{1}{2} \left(\frac{\pi}{2} - \alpha - \frac{1}{\tan \alpha} \right) \right] = \frac{1}{2} \left(\frac{\pi}{2} - \pi - \frac{1}{\tan \pi^-} \right) = \frac{1}{2} \left(-\frac{\pi}{2} - \frac{1}{0^-} \right) = \\ = \frac{1}{2} \left(-\frac{\pi}{2} + \infty \right) = +\infty$$

Quando α tende para π^- , o comprimento do segmento $[BC]$ tende para $+\infty$ e, portanto, a área da região sombreada tende para $+\infty$.

6.3.

$$A'(\alpha) = \frac{1}{2} \left(0 - 1 - \frac{0 - 1 \times (\tan \alpha)'}{\tan^2 \alpha} \right) = \frac{1}{2} \left(-1 + \frac{1}{\cos^2 \alpha} \right) = \frac{1}{2} \left(-1 + \frac{1}{\sin^2 \alpha} \right)$$

$$A'(\alpha) = 0 \Leftrightarrow \frac{1}{2} \left(-1 + \frac{1}{\sin^2 \alpha} \right) = 0 \Leftrightarrow \frac{1}{\sin^2 \alpha} = 1 \Leftrightarrow \sin \alpha = \pm 1 \wedge \sin \alpha \neq 0 \Leftrightarrow \\ \alpha = \frac{\pi}{2} + k\pi \wedge \alpha \neq k\pi, k \in \mathbb{Z}$$

Não existem soluções em $\left] \frac{\pi}{2}, \pi \right[$.

α	$\frac{\pi}{2}$		π
$A'(\alpha)$	+		
A		↗	

Portanto A é estritamente crescente em $\left] \frac{\pi}{2}, \pi \right[$.